

Measurements on a subcritical reactor using a pulsed neutron source¹

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With 4 figures in the text

Summary

A pulsed neutron source was introduced into the Swedish heavy-water reactor R1 when it was in a subcritical state. The time behaviour of the neutron density was studied by means of a multichannel time analyser. Using this technique the time constant of the prompt neutrons and the reactivity was determined at several heavy water levels down to about 20 cm below the critical level. It was found that the reactivity equivalent of the heavy water is $(7.2 \pm 0.1) \times 10^{-5}$ per mm, and that the mean lifetime of the thermal neutrons in the reactor is 0.68 ± 0.02 msec. These values refer to the critical level and are based on a fraction of delayed and photo neutrons amounting to 0.78×10^{-2} . A calibration of the safety rods gave the result that each of them corresponds to a reactivity of 1.7 ± 0.2 per cent and both together to 3.4 ± 0.4 per cent.

Introduction

Among the quantities which are of fundamental importance to know for the operation of a reactor are the reactivity equivalents of the safety and control rods. In a heavy-water reactor the dependence of the reactivity on the amount of heavy water is also of great importance.

For the determination of reactivities of this kind two methods are generally used. The first method is to measure the period of the supercritical reactor and calculate the excess reactivity with Nordheim's formula [1, 2, 3]. In the second method the influence of different absorbers on the multiplication of the subcritical reactor is studied by measuring the neutron density with a counter [3, 4]. The advantage is that large reactivities can be measured without the hazards involved in having the reactor supercritical with a large excess reactivity, but the method gives only relative values. In order to obtain the absolute reactivities at least one measurement according to the first method must be made.

The mean lifetime of the thermal neutrons in a reactor is of interest for the kinetic behaviour and for estimating the source strength of the reactor. It can be determined by oscillating an absorber in the reactor and measuring the phase and amplitude of

¹ A preliminary report on this work was given at the Geneva Conference, paper P/789 (1955).

the variations caused in the neutron density as a function of the frequency [5, 6, 7, 8]. Another method is to investigate the time behaviour of the neutron density after a step change in the reactivity [8, 9].

There is, however, a method (suggested by G. von Dardel) by which it is possible to determine both reactivities and the neutron lifetime from measurements on a subcritical reactor. In this method the reactor is pulsed with a neutron source and the time behaviour of the neutron density is studied. In the present paper such experiments with the Swedish heavy-water reactor R1 will be described. Similar experiments have independently been made in Russia on uranium-graphite lattices [10].

Theory

Suppose that a source inside a subcritical reactor produces rectangular neutron pulses, each containing S neutrons. It is assumed for simplification of the treatment that the repetition time is much shorter than the shortest periods of the delayed neutrons and that the time constant of the reactor is much shorter than the pulse length. The neutron density in the reactor will then vary with time as shown in Fig. 1. The prompt multiplication gives rise to pulses of the same shape as the source pulses. In addition there is a constant density due to the delayed neutrons, which cannot follow the rapid variations of the source. The area of a pulse above the constant value in Fig. 1 is denoted by A_1 and the area under the delayed neutron curve during one period by A_2 . The prompt multiplication and the total multiplication respectively correspond to [11]

$$A_1 = \frac{a \cdot S}{1 - k_{\text{eff}}(1 - \beta)} \quad (1)$$

$$A_1 + A_2 = \frac{a \cdot S}{1 - k_{\text{eff}}} \quad (2)$$

where β is the fraction of delayed neutrons (for a heavy-water reactor including photo neutrons) and a is a constant. Equations (1) and (2) give directly

$$\frac{A_1}{A_2} = \frac{1 - k_{\text{eff}}}{\beta k_{\text{eff}}} = \frac{\rho}{\beta} \quad (3)$$

where ρ , the *negative* reactivity, is defined as

$$\rho = \frac{1 - k_{\text{eff}}}{k_{\text{eff}}} \quad (4)$$

If the fraction of delayed neutrons is known, it will thus be possible to determine the reactivity by a measurement with a pulsed neutron source.

In the case actually considered the simplifying assumptions made above are not valid. The finite value of the reactor time constant, l_p , and the short periods of delayed neutrons modify the curve to a shape shown in Fig. 2. However, the area under the curve can still be divided into two parts, one arising from the prompt and one from the delayed neutrons. It is shown in Appendix I that with the usual approximations for a bare thermal reactor [11], assuming the age of the delayed neutrons to be the

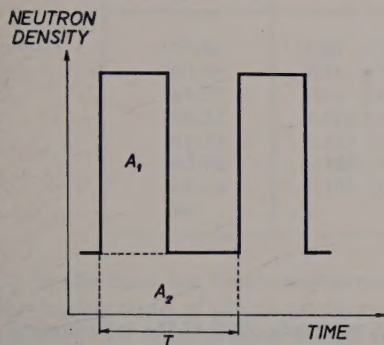


Fig. 1. The neutron density in a pulsed subcritical reactor as a function of time under the simplifying conditions given in the text.

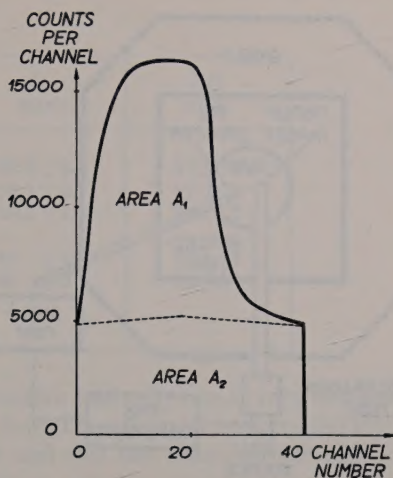


Fig. 2. A measured curve obtained with the reactor subcritical by about one per cent. The pulse length was 0.25 sec and the repetition time 0.5 sec. Measuring time was one hour. The dotted line indicates the neutron density arising from delayed neutrons. According to eq. (3), $A_1/A_2 = \rho/\beta$.

same as the age of the prompt neutrons, the ratio of areas is still given by eq. (3). The two-group theory for a reactor with reflector also yields the same result.

The rise and decay time of the neutron density is determined by the reactor time constant l_p . From eq. (A1) in Appendix I we see that

$$l_p = \frac{l}{1 - (1 - \beta) k_{\text{eff}}} = \frac{l(1 + \rho)}{\beta + \rho} \quad (5)$$

where l denotes the mean lifetime of the thermal neutrons in the reactor. By measuring the time constant and using the reactivity value obtained from eq. (3) the mean lifetime can thus be determined.

Experimental arrangement

The measurements were made with the Swedish heavy-water reactor R1 in a subcritical state [12]. A schematic view of the experiment is seen in Fig. 3. The neutron pulses were produced by means of a 150 kV acceleration tube [13]. It was placed horizontally in front of the reactor shield, and a 4 m long aluminium tube, 5 cm in diameter, was extended into the reactor tank through a horizontal channel. A zirconium-tritium target was attached to the end of the tube. Deuterium ions striking the target produced 14 MeV neutrons, by the T-D reaction. By modulation of the ion source [14] a pulsed neutron source was thus obtained about 30 cm from the centre of the reactor.

For convenient control of the ion beam it was passed through a hole in a mirror at 45° to the beam direction. The beam was strongly defocused during its passage

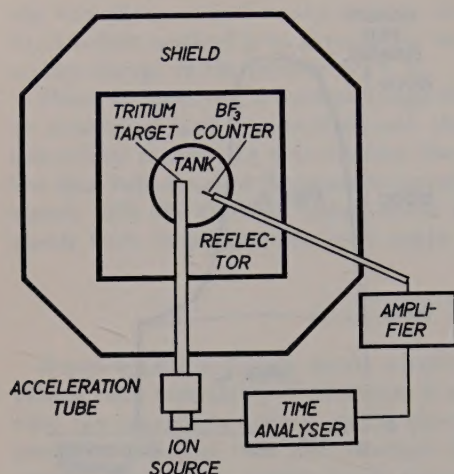


Fig. 3. A schematic view of the experimental arrangement.

to the target, because the pumping velocity in the target tube was rather low. The tube walls received about as large an ion current as the target and this gave rise to unwanted D-D neutron sources. However, the broadness of the beam had the advantage that higher current could be allowed to reach the delicate target material without any risk of damaging it.

In another horizontal channel which is about 20 cm lower than the target channel, a small BF_3 proportional counter mounted on a 4 m probe was introduced. The sensitive volume was 2.5 cm^3 and the counter was filled with 50 mm BF_3 and argon to a total pressure of 700 mm Hg. After amplification the pulses from the counter were fed into a 40-channel time analyser [15], by which the time behaviour of the neutron density in the reactor could be studied.

Channel widths of the order of 0.01 sec were used for the measurements. They were obtained by feeding the pulses from the second frequency divider through an external scaler to the gate generators. The curve shown in Fig. 2, for example, was taken with a channel width of $4 \times 12 \times 256 = 12288 \mu\text{sec}$.

The first 20 channels of the time analyser feed scale-of-16 units and the other 20 feed scale-of-4 units. With the present channel widths the counting rate which could be used was limited by the mechanical registers connected to the scalers. To avoid counting losses in the registers the time analyser was coupled so that only every second channel was used, but with a scale-of-64 reduction.

Reactivity measurements

For the determination of the reactivity equivalent of the heavy water, measurements were made at 7 different water levels down to 18 cm below the critical level. All safety and control rods were out of the reactor. The pulse repetition time was 0.5 or 1 sec and the pulse length half this period. The target current during the pulse was between 10 and $20 \mu\text{A}$. The neutron production was always started at least 10 min before the measurement commenced in order to reach equilibrium as regards the delayed neutrons.

Table 1.

H cm	ϱ/β	l_p msec	l/β msec
177.38	1.89 ± 0.05	30.3 ± 0.5	87 ± 2
182.48	1.31 ± 0.03	38.1 ± 0.7	87 ± 2
187.27	0.78 ± 0.02	50.2 ± 1.2	89 ± 2
189.22	0.566 ± 0.016		
191.22	0.353 ± 0.012	63 ± 4	85 ± 6
192.73	0.192 ± 0.011	65 ± 8	78 ± 11
193.76	0.105 ± 0.010		
195	0		

Up to the time for these measurements the reactor had only run at very low power, so the background of photo neutrons was very low. The observed background mainly arose from the spontaneous fission of uranium and was less than one per cent even during the off-time of the pulse.

The heavy-water level was read on the gauge glass in the control room simultaneously with the temperature of the gauge glass and of the uranium rods. Corrections were made for the differences in temperature between the measurements.

The critical heavy-water level was determined in the following way: A control rod was put into the reactor, the water was pumped up above the expected level and the reactor was run up to a power of a few watts. The water level was then lowered and the power kept constant by means of the control rod. This was continued until the control rod was completely out of the reactor. By small adjustments of the water the critical level could then be determined as accurately as a reading could be made on the gauge glass, that is ± 0.2 mm.

When the reactivity is to be calculated from a measured curve like the one shown in Fig. 2, some uncertainty may arise depending upon how the area is divided into the two parts A_1 and A_2 . This has been avoided by using eq. (A12) of Appendix II, where directly measured values can be used in the numerical computation in both numerator and denominator. The figures used for the delayed neutron yields and periods are taken from Hughes *et al.* [16] and Bendt and Scott [17], and photo neutron data from Johns and Sargent [18]. The total fraction of delayed and photo neutrons was then 0.78 per cent. All groups of delayed and photo neutrons were assumed to have an energy of 0.5 MeV. By using eq. (A6) of Appendix I corrections were made for the difference in age between the prompt and delayed neutrons. An age difference of 9 cm² was assumed and the correction amounted to about 1 per cent for the largest reactivity and about 8 per cent for the smallest. In Table 1 the ϱ/β values obtained from the measurements are listed together with estimated uncertainties.

The influence of higher harmonic modes

When evaluating the reactivities, corrections also had to be applied for the presence of higher harmonic modes in the spatial neutron distribution. This has been done by expanding the distribution of source neutrons in harmonic components and using eq. (A26) in Appendix III. Since the target is located on one side of the reactor centre and the detector on the opposite side, the most serious harmonic mode is the one with a single radial nodal plane through the cylinder axis. This mode has a

Table 2.

Distance from reactor centre cm	ϱ/β	l_p msec
44	0.89	49.1
68	0.83	46.1
87	0.85	47.7
111	0.92	50.1
136	0.86	47.3
	each ± 0.02	each ± 1.4

negative amplitude at the detector position, which means that a positive correction has to be applied to the reactivity. As an example, it may be mentioned that for the measurement shown in Fig. 2 a correction of two per cent was calculated.

We also tried to study the influence of the harmonic modes by reactivity measurements in five different positions of the detector from close to the centre of the tank to the middle of the reflector. The results are given in Table 2. They have not been corrected for the higher harmonics, but the influence of the detector on the reactivity has been taken into account. (This was investigated with the pile running at a few watts, and the corrections were of the order of one per cent.)

The scattering in the five values is greater than would be expected statistically. The large difference between the first value and the two subsequent ones can largely be ascribed to the most pronounced harmonic mode. However, the harmonic modes have a greater influence here than in the ordinary reactivity measurements, because the pulse length for these measurements was only one tenth of the repetition time, a circumstance which favours the higher harmonics.

Uncertainties in the reactivity measurements

To illustrate the uncertainties involved in the measurements we consider the case illustrated in Fig. 2, which, according to Table 1, gave the result $\varrho/\beta = 1.31 \pm 0.03$.

The statistical errors from the points in the curve give rise to an error of 0.2 per cent in the numerator and 0.8 per cent in the denominator of the right-hand side of eq. (A12). The periods and relative yields of the delayed neutrons enter in a similar way into both the numerator and the denominator and will therefore have a very small effect. We assume 0.2 per cent for the periods and 0.2 per cent for the yield, which means that the short periods could be incorrect by 10 per cent. The assumption of equal ages for all delayed and photo neutrons can at most give rise to 0.2 per cent error. This results in a 0.9 per cent error in $(\varrho/\beta + 1)$ or 1.6 per cent in ϱ/β . A 20 per cent uncertainty assumed in the correction for different ages between the prompt and delayed neutrons gives an error of only 0.2 per cent. The uncertainty due to the harmonic modes is estimated to be as large as the correction, namely 2 per cent. The error caused by non-saturation of the delayed neutron activities before measurements commenced could at a maximum amount to 0.5 per cent. The D-D neutron sources and variations in the source strength during the measurements do not introduce more than 0.1 per cent each. A total error of 2.6 per cent is obtained for ϱ/β .

If the absolute value of the reactivity is calculated we have in addition the uncertainty in β , which must be considered to be as large as 6 per cent.

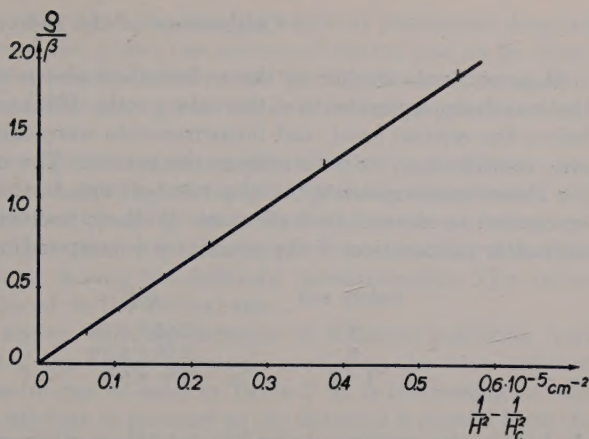


Fig. 4. The reactivity as a function of $1/H^2 - 1/H_c^2$. The straight line was fitted to the measured points by the method of least squares.

The reactivity equivalent of the heavy water

According to the one-group theory for a bare reactor we can write

$$\rho = \frac{1 - k_{\text{eff}}}{k_{\text{eff}}} = \frac{\pi^2 M^2}{1 + M^2 B_c^2} \left(\frac{1}{H^2} - \frac{1}{H_c^2} \right) \quad (6)$$

where H is the effective height of the reactor and the index c denotes the value at criticality.

In Fig. 4 the ρ/β values obtained from the measurements are plotted against $1/H^2 - 1/H_c^2$. The effective critical height, H_c , was assumed to be 195 cm according to the measurements by Larsson [3], which gave $H_c = 195_{-5}^{+15}$. Theoretical work has given 200 cm [3].

The points in Fig. 4 fit a straight line very well, and from the line we find the reactivity equivalent of the heavy water to be $(7.2 \pm 0.1) \times 10^{-5}$ per mm D_2O at the critical level. This is in agreement with supercritical measurements by G. Jonsson [3], who obtained $(7.0 \pm 0.1) \times 10^{-5}$. The errors do not include the 6 per cent uncertainty in the total yield of delayed and photo neutrons.

From the slope of the line we find $M^2 = 336 \text{ cm}^2$, which at first seems too large. For the reactor lattice we can calculate $M^2 = 235$ from the measurements by Cohen [19]. According to calculations by Pershagen [3], the presence of reactor channels increases this by about 8.6 per cent. The presence of the reflector, not taken into account in eq. (6), raises the effective M^2 value by a further 25 per cent approximately [20]. We thus arrive at about 320 cm^2 . The remaining discrepancy can easily be explained by the uncertainty in the effective height of the reactor. While this does not have any influence on the reactivity equivalent of the water, an error of one per cent in the effective height will lead to as much as three per cent error in M^2 . By assuming the critical height to be 192 cm the M^2 value can thus be reduced to about 320 cm^2 . The critical height is then still in good agreement with the experimental value.

Calibration of the safety rods

Measurements similar to those described above were made in order to determine the reactivity equivalents of the safety rods. The heavy-water level was then 1.25 cm below the critical level, and measurements were carried out with safety rod number one, number two, or both rods in the reactor. The background was relatively higher for these measurements, as the reactor was farther from criticality, but it never amounted to more than 5 per cent. With no corrections for the harmonic modes we find after subtraction of the reactivity corresponding to the 1.25 cm of heavy water:

Safety rod	ρ/β	ρ (per cent)
1	2.02 ± 0.10	1.58 ± 0.08
2	2.06 ± 0.10	1.61 ± 0.08
1 + 2	3.9 ± 0.3	3.1 ± 0.2

As before, the uncertainty in β is not taken into account in the errors given.

On another occasion, when the uranium lattice in the reactor was not quite complete, a few more measurements were made with the detector in two different positions. In the "in" position, which was the same as that used previously, the detector was only about 30 cm from safety rod number one, while in all other cases the distance from the detector to the safety rods was at least 60 cm. The results are:

Safety rod	ρ/β with the detector	
	in	out
1	2.22 ± 0.06	2.00 ± 0.07
2	1.99 ± 0.06	2.02 ± 0.07

These results do not indicate any strong dependence on the detector position. Because the neutron distribution is much disturbed by the safety rods it is difficult to calculate the harmonic corrections in this case. The harmonic mode having one single radial nodal plane through the cylinder axis will still be the most serious one, however. This means that a positive correction has to be applied to the reactivities. We assume 5 per cent for each safety rod and 10 per cent for both together. The final values are then:

Safety rod	Reactivity (per cent)
1	1.7 ± 0.2
2	1.7 ± 0.2
1 + 2	3.4 ± 0.4

The results are in good agreement with the measurements by G. Jonsson [3].

Determination of the neutron lifetime

In order to determine the mean lifetime of the thermal neutrons in the reactor a series of measurements was made with the pulse length only one tenth of the repetition time, which was 0.25 or 0.5 sec. With this arrangement it was possible to follow the decay of the neutron density much more closely. These experiments were done at five of the heavy-water levels used previously.

As the repetition time was so long that the delayed neutron precursors decayed considerably during the off-time of the pulse, the measured curves had to be fitted to a sum of two exponentials. One corresponded to the prompt neutron decay, and one to that of the delayed neutrons. The fitting was done with a least-squares method, and the results are given in Table 1.

The time constant l_p of the prompt neutrons is related to the mean lifetime l of the thermal neutrons in the reactor according to eq. (5). By using this relation and the measured ratio ρ/β the values for l/β also given in Table 1 have been calculated. A small correction, less than 0.5 per cent, has been applied to eliminate the effect of the different sizes of the reactor during the different measurements. The values given are the mean lifetimes reduced to the critical size.

The measurements described above, with the detector in different positions, were also used to study the influence of the higher harmonic modes on the time constant of the prompt neutrons. The results can be seen in Table 2. It is interesting to note that there is no tendency for the lifetime to increase as the detector is moved from the reactor centre to the reflector, a tendency of which Hellstrand [8] found a vague indication. In fact the spread between the values is not inconsistent with the 3 per cent statistical uncertainty in each measurement. It therefore seems as if the higher harmonic modes have a very small influence, which is also confirmed by the fact that the measurements of the mean lifetime made at different reactivities agree within the limits of error.

As a weighted mean we get for l/β a value of 87 msec. Assuming $\beta = 0.78 \times 10^{-2}$ we find a mean lifetime of 0.68 ± 0.02 msec. The error here takes account of the uncertainty due to the harmonic modes but not of the 6 per cent uncertainty in β . The result agrees very well with the 0.71 ± 0.03 msec obtained by Hellstrand [8], who used the slightly different $\beta = 0.79 \times 10^{-2}$.

In this connection it may be pointed out how desirable it would be to have more accurate determinations of the total yield of the delayed and photo neutrons, β . The large uncertainty in this parameter enters into the absolute values of all the measured quantities in this paper.

Conclusions

The present investigation shows that the pulsed neutron source is a powerful means of studying a subcritical reactor. In addition to the measurements described above, the method can of course also be used for determination of the critical size of a subcritical assembly. As the uncertainties in the reactivity measurements described above are mainly of systematic type, it seems possible to increase the accuracy considerably. The most important thing would be to suppress the higher harmonic modes by a suitable arrangement of the source and the detector. With this improvement it would also be possible to make the measurements farther from criticality. It is therefore considered that the pulsed neutron source method applied to a subcritical assembly may be a valuable complement to exponential experiments.

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APPENDIX I

We consider a bare subcritical reactor which contains a neutron source. The source strength is assumed to be a periodic function of time, $S(t)$. It is further supposed that the reactor is so close to criticality that only the fundamental mode of the spatial neutron distribution is important. According to ref. [11], p. 295 ff. and with the same notation we can then write:

$$l \frac{\partial \phi}{\partial t} = -[1 - (1 - \beta) k_{\text{eff}}] \phi + \frac{p \sum_i e^{-B^2 \tau_i} \lambda_i C_i}{\sum_a (1 + B^2 L^2)} + \frac{p e^{-B^2 \tau_s} S}{\sum_a (1 + B^2 L^2)}, \quad (\text{A } 1)$$

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \frac{k}{p} \beta_i \sum_a \phi. \quad (\text{A } 2)$$

The ages of the prompt neutrons, the source neutrons, and the i th group of delayed neutrons are called τ_p , τ_s , and τ_i respectively. As $S(t)$ is a periodic function, ϕ and C_i must also be periodic functions when equilibrium has been reached for the delayed neutrons. When eqs. (A1) and (A2) are integrated over one period T , the terms on the left side therefore disappear and we obtain

$$[1 - (1 - \beta) k_{\text{eff}}] \int_0^T \phi dt = \frac{p}{\sum_a (1 + B^2 L^2)} \left[\sum_i e^{-B^2 \tau_i} \int_0^T \lambda_i C_i dt + e^{-B^2 \tau_s} \int_0^T S dt \right], \quad (\text{A } 3)$$

$$\int_0^T \lambda_i C_i dt = \frac{k}{p} \beta_i \sum_a \int_0^T \phi dt. \quad (\text{A } 4)$$

By summing all the groups of delayed neutrons we get from eq. (A4):

$$\sum_i e^{-B^2 \tau_i} \int_0^T \lambda_i C_i dt = \frac{k}{p} \sum_a \int_0^T \phi dt \sum_i \beta_i e^{-B^2 \tau_i}. \quad (\text{A } 5)$$

Now $\int_0^T \phi dt$ can be eliminated between eqs. (A3) and (A5). The result is:

$$\frac{A_1}{A_2} = \frac{e^{-B^2 \tau_s} \int_0^T S dt}{\sum_i e^{-B^2 \tau_i} \int_0^T \lambda_i C_i dt} = \frac{1 - (1 - \beta) k_{\text{eff}} - k_{\text{eff}} e^{B^2 \tau_p} \sum_i \beta_i e^{-B^2 \tau_i}}{k_{\text{eff}} e^{B^2 \tau_p} \sum_i \beta_i e^{-B^2 \tau_i}}. \quad (\text{A } 6)$$

In the case when all groups of the delayed neutrons have the same age as the prompt neutrons, this expression reduces to

$$\frac{A_1}{A_2} = \frac{1 - k_{\text{eff}}}{\beta k_{\text{eff}}} = \frac{\rho}{\beta}. \quad (\text{A } 7) = (3)$$

APPENDIX II

If eq. (A2) is multiplied by $\exp(\lambda_i t)$ and integrated over the period T , we obtain

$$C_i(T) e^{\lambda_i T} - C_i(0) = \frac{k}{p} \beta_i \Sigma_a \int_0^T \phi e^{\lambda_i t} dt. \quad (\text{A } 8)$$

As C_i is a periodic function, $C_i(T) = C_i(0)$, and thus

$$C_i(0) = \frac{k \beta_i \Sigma_a}{p(e^{\lambda_i T} - 1)} \int_0^T \phi e^{\lambda_i t} dt. \quad (\text{A } 9)$$

Multiplying by $\exp(-B^2 \tau_i) \lambda_i$ and summing over the delayed neutron groups, we get

$$\sum_i e^{-B^2 \tau_i} \lambda_i C_i(0) = \frac{k \Sigma_a}{p} \sum_i \frac{\beta_i \lambda_i e^{-B^2 \tau_i}}{e^{\lambda_i T} - 1} \int_0^T \phi e^{\lambda_i t} dt. \quad (\text{A } 10)$$

Combination with eq. (A5) gives

$$\sum_i e^{-B^2 \tau_i} \int_0^T \lambda_i C_i dt = \frac{\sum_i e^{-B^2 \tau_i} \lambda_i C_i(0) \int_0^T \phi dt \sum_i \beta_i e^{-B^2 \tau_i}}{\sum_i \beta_i \lambda_i \frac{e^{-B^2 \tau_i}}{e^{\lambda_i T} - 1} \int_0^T \phi e^{\lambda_i t} dt}. \quad (\text{A } 11)$$

By substitution in eq. (A3) we can obtain a corresponding expression for $\exp(-B^2 \tau_s) \int_0^T S dt$. The ratio is

$$\frac{A_1}{A_2} = \frac{e^{-B^2 \tau_s} \int_0^T S dt}{\sum_i e^{-B^2 \tau_i} \int_0^T \lambda_i C_i dt} = \frac{\sum_i \beta_i \lambda_i \frac{e^{-B^2 \tau_i}}{e^{\lambda_i T} - 1} \int_0^T \phi e^{\lambda_i t} dt}{\frac{p}{[1 - (1 - \beta) k_{\text{eff}}] \Sigma_a (1 + B^2 L^2)} \sum_i e^{-B^2 \tau_i} \lambda_i C_i(0) \sum_i \beta_i e^{-B^2 \tau_i}} - 1. \quad (\text{A } 12)$$

APPENDIX III

If the treatment in chapter 7 of ref. [11] is extended to include one group of delayed neutrons, we get the following equations

$$\frac{1}{v} \frac{\partial \phi(r, t)}{\partial t} = D \nabla^2 \phi(r, t) - \Sigma_a \phi(r, t) + p q(r, \tau_{th}, t) \quad (\text{A } 13)$$

$$\nabla^2 q(r, \tau, t) = \frac{\partial q(r, \tau, t)}{\partial \tau} \quad (\text{A } 14)$$

$$\frac{\partial C(r, t)}{\partial t} = -\lambda C(r, t) + \frac{k}{p} \beta \Sigma_a \phi(r, t) \quad (\text{A } 15)$$

$$q(r, 0, t) = \frac{k}{p} \Sigma_a (1 - \beta) \phi(r, t) + \lambda C(r, t) + S(r, t). \quad (\text{A } 16)$$

We have here assumed that the prompt, delayed, and source neutrons have the same energy. By separating the variables in eq. (A 14) and introducing the eigenfunctions obtained from the boundary conditions we get

$$q(r, \tau, t) = \sum_{n=1}^{\infty} K_n R_n(r) e^{-B_n^2 \tau} T_n(t) \quad (\text{A } 17)$$

$$S(r, t) = \sum_{n=1}^{\infty} S_n(t) R_n(r) \quad (\text{A } 18)$$

$$C(r, t) = \sum_{n=1}^{\infty} C_n(t) R_n(r) \quad (\text{A } 19)$$

$$\phi(r, t) = \frac{p}{k \Sigma_a (1 - \beta)} \sum_{n=1}^{\infty} \{K_n T_n(t) - \lambda C_n(t) - S_n(t)\} R_n(r). \quad (\text{A } 20)$$

These expansions are put into eqs. (A 13) and (A 15). Integrating over one period, T , and simplifying yields the following relations:

$$\int_0^T S_n dt = (1 - k_n) K_n \int_0^T T_n dt, \quad (\text{A } 21)$$

$$\int_0^T \lambda C_n dt = \beta k_n K_n \int_0^T T_n dt. \quad (\text{A } 22)$$

By integration of eq. (A 20) over one period, and using the above expressions, we get

$$\int_0^T \phi(r, t) dt = \frac{p}{k \sum_{n=1}^{\infty} k_n K_n R_n(r)} \int_0^T T_n dt. \quad (\text{A } 23)$$

If now eqs. (A21) and (A22) are added and $K_n \int_0^T T_n dt$ is eliminated, we find the following expression for the area under a measured curve:

$$\int_0^T \phi(r, t) dt = \frac{p}{k \sum_{n=1}^{\infty} \frac{k_n}{1 - k_n(1 - \beta)}} \left\{ \int_0^T \lambda C_n dt + \int_0^T S_n dt \right\} R_n(r). \quad (\text{A } 24)$$

The ratio of the two parts of the area is then

$$\frac{A_1}{A_2} = \frac{\sum_{n=1}^{\infty} \frac{k_n}{1 - k_n(1 - \beta)} R_n(r) \int_0^T S_n dt}{\sum_{n=1}^{\infty} \frac{k_n}{1 - k_n(1 - \beta)} R_n(r) \int_0^T \lambda C_n dt}. \quad (\text{A } 25)$$

This is finally transformed to

$$\frac{A_1}{A_2} = \frac{\rho}{\beta} \frac{1 + \sum_{n=2}^{\infty} \frac{k_n}{k_1} \frac{1 - k_1(1 - \beta)}{1 - k_n(1 - \beta)} \frac{R_n(r)}{R_1(r)} \frac{\int_0^T S_n dt}{\int_0^T S_1 dt}}{1 + \sum_{n=2}^{\infty} \frac{k_n^2}{k_1^2} \frac{1 - k_1}{1 - k_n} \frac{1 - k_1(1 - \beta)}{1 - k_n(1 - \beta)} \frac{R_n(r)}{R_1(r)} \frac{\int_0^T S_n dt}{\int_0^T S_1 dt}}. \quad (\text{A } 26)$$

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